

# TEAM-Math and AMSTI Professional Mathematics Learning Communities

## Effective Questioning -- Discourse

### Vignette for 6-8

#### Ratios, Proportions, and Proportional Reasoning

It is often tempting to provide too much help when students struggle with ratio and proportion problems, thus removing their opportunities for genuine engagement with the mathematics. For example, consider the following problem:

Heart A beats 15 times in 8 seconds, and heart B beats 38 times in 20 seconds. Which heart is beating faster, A or B, or do they both beat at the same pace?

The discussion that follows presents two different dialogues between a teacher and a student working on the problem. The dialogues are based on an unpublished, small-scale study and reflect the general tone and substance of the interactions, although they are not direct transcripts.

*Student:* I don't know how to solve this problem. I'm stuck.

*Teacher:* How can you compare two heart rates?

*Student:* I don't know.

*Teacher:* Think of other rates you've heard of before. What are some rates you know of?

*Student:* Well ... there's miles per hour.

*Teacher:* Good. What makes that a rate?

*Student:* Because it has a *per*.

*Teacher:* Another way of saying that is because it's "per one." How many miles you travel in one hour. So how can you turn this into a rate?

*Student:* Find a "per one"?

*Teacher:* Good. So do you think it should be beats per minute or minutes per beat?

*Student:* Beats per minute.

*Teacher:* Okay, so divide to see how many beats per minute each heart will give you.

Reflect 3.2 asks you to evaluate the teacher's handling of the various "teachable moments" in this scenario.

#### Reflect 3.2

Consider the dialogue above in which the student struggles to determine which heart is beating faster.

a. Do you think the teacher provides the student with sufficiently rich problem-solving opportunities?

b. In what ways might you intervene differently with your own students?

Do you think the teacher helps the student too much? Are you nevertheless unsure about how you could manage the dialogue differently? Consider the following alternative scenario, in which the teacher scaffolds the student's thinking but deliberately avoids solving the problem for her:

*Student:* I don't know how to solve this problem. I'm stuck.

*Teacher:* What's the problem asking you to do?

*Student:* Figure out which heart is faster.

*Teacher:* Okay, so what does "faster" mean to you?

*Student:* Whichever heart beats quicker ... has more beats.

*Teacher:* Okay, ... so does that help you answer the question?

*Student:* Well, B gives 38 beats, but it takes more time—20 seconds instead of 8.

*Teacher:* Why does that matter?

*Student:* How many seconds it takes matters too. That's why I'm stuck.

*Teacher:* Ah, okay. So beats *and* seconds matter?

*Student:* Yeah.

*Teacher:* So you have A that gives 15 beats in 8 seconds, and B that gives 38 beats in 20 seconds. Can you figure out a way to compare those two?

*Student:* I can't really do it, because even though 38 is more than 15, 20 is also more than 8.

*Teacher:* Let me give you a different problem. Say that A gives 15 beats in 8 seconds, but B gives 20 beats in 16 seconds. Which heart beats faster?

*Student:* Hmm... [Starts to write].

*Teacher:* I'll leave you to work on this for a while.

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*Teacher:* So what did you figure out?

*Student:* Well, this one was pretty easy because if A beats 15 in 8 seconds, you can just multiply that by 2 and get 30 beats in 16 seconds. That's more than 20 beats in 16 seconds, so A has to be faster.

*Teacher:* Okay, so it looks like you multiplied A by 2. Why did you do that?

*Student:* Because then I have the same number of seconds.

*Teacher:* Why is that important?

*Student:* Because if the time is the same, then I can just see which heart had more beats in the same amount of time.

*Teacher:* So is there a way you could do that to solve the original problem?

*Student:* Make the time the same? I don't know, because the numbers don't work out very well.

*Teacher:* Why don't you try it out—see what you can come up with. I'll come back in a little while to see how you're doing.