Principles to Actions

Ensuring Mathematical Success for All

NCTM | NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

Facilitate Meaningful Mathematical Discourse

Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.

Effective mathematics teaching engages students in discourse to advance the mathematical learning of the whole class. Mathematical discourse includes the purposeful exchange of ideas through classroom discussion, as well as through other forms of verbal, visual, and written communication. The discourse in the mathematics classroom gives students opportunities to share ideas and clarify understandings, construct convincing arguments regarding why and how things work, develop a language for expressing mathematical ideas, and learn to see things from other perspectives (NCTM 1991, 2000).

Discussion

Discourse that focuses on tasks that promote reasoning and problem solving is a primary mechanism for developing conceptual understanding and meaningful learning of mathematics (Michaels, O'Connor, and Resnick 2008). According to Carpenter, Franke, and Levi (2003, p. 6),

> Students who learn to articulate and justify their own mathematical ideas, reason through their own and others' mathematical explanations, and provide a rationale for their answers develop a deep understanding that is critical to their future success in mathematics and related fields.

Although discourse provides important opportunities for students to learn what mathematics is *and* how one does it, creating a culture of discourse in the mathematics classroom also presents challenges. Teachers must determine how to build on and honor student thinking while ensuring that the mathematical ideas at the heart of the lesson remain prominent in class discussions (Engle and Conant 2002). For example, in orchestrating a class discussion of student approaches to solving a task, the teacher must decide what approaches to share, the order in which they should be shared, and the questions that will help students make connections among the different strategies and the key disciplinary ideas that are driving the lesson. Such discussions can easily become little more than elaborate show-and-tell sessions (Wood and Turner-Vorbeck 2001) in which it is not clear what each solution adds to students' developing understanding or how it advances the mathematical storyline of the lesson. Smith and Stein (2011) describe five practices for effectively using student responses in whole-class discussions:

1. *Anticipating* student responses prior to the lesson
2. *Monitoring* students' work on and engagement with the tasks
3. *Selecting* particular students to present their mathematical work
4. *Sequencing* students' responses in a specific order for discussion
5. *Connecting* different students' responses and connecting the responses to key mathematical ideas

Students must also have opportunities to talk with, respond to, and question one another as part of the discourse community, in ways that support the mathematics learning of all students in the class. Hufferd-Ackles, Fuson, and Sherin (2004) describe a framework for moving toward a classroom community centered on discourse. They examine how teachers and students proceed through levels in shifting from a classroom in which teachers play the leading role in pursuing student mathematical thinking to one in which they assist students in taking on important roles. The framework describes growth in five components (Hufferd-Ackles, Fuson, and Sherin 2014):

1. How the teacher supports student engagement
2. Who serves as the questioner and what kinds of questions are posed
3. Who provides what kinds of explanations
4. How mathematical representations are used
5. How much responsibility students share for the learning of their peers and themselves

Figure 11 shows a table developed by Hufford-Ackles, Fuson, and Sherin (2014) to describe the levels of classroom discourse through which teachers and their students advance.

	Teacher role	**Questioning**	**Explaining mathematical thinking**	**Mathematical representations**	**Building student responsibility within the community**
Level 0	Teacher is at the front of the room and dominates conversation.	Teacher is only questioner. Questions serve to keep students listening to teacher. Students give short answers and respond to teacher only.	Teacher questions focus on correctness. Students provide short answer-focused responses. Teacher may give answers.	Representations are missing, or teacher shows them to students.	Culture supports students keeping ideas to themselves or just providing answers when asked.
Level 1	Teacher encourages the sharing of math ideas and directs speaker to talk to the class, not to the teacher only.	Teacher questions begin to focus on student thinking and less on answers. Only teacher asks questions.	Teacher probes student thinking somewhat. One or two strategies may be elicited. Teacher may fill in an explanation. Students provide brief descriptions of their thinking in response to teacher probing.	Students learn to create math drawings to depict their mathematical thinking.	Students believe that their ideas are accepted by the classroom community. They begin to listen to one another supportively and to restate in their own words what another student has said.
Level 2	Teacher facilitates conversation between students, and encourages students to ask questions of one another.	Teacher asks probing questions and facilitates some student-to-student talk. Students ask questions of one another with prompting from teacher.	Teacher probes more deeply to learn about student thinking. Teacher elicits multiple strategies. Students respond to teacher probing and volunteer their thinking. Students begin to defend their answers.	Students label their math drawings so that others are able to follow their mathematical thinking.	Students believe that they are math learners and that their ideas and the ideas of their classmates are important. They listen actively so that they can contribute significantly.
Level 3	Students carry the conversation themselves. Teacher only guides from the periphery of the conversation. Teacher waits for students to clarify thinking of others.	Student-to-student talk is student initiated. Students ask questions and listen to responses. Many questions ask "why" and call for justification. Teacher questions may still guide discourse.	Teacher follows student explanations closely. Teacher asks students to contrast strategies. Students defend and justify their answers with little prompting from the teacher.	Students follow and help shape the descriptions of others' math thinking through math drawings and may suggest edits in others' math drawings.	Students believe that they are math leaders and can help shape the thinking of others. They help shape others' math thinking in supportive, collegial ways and accept the same support from others.

Fig. 11. Levels of classroom discourse. From Hufford-Ackles, Fuson, and Sherin (2014), table 1.

Teacher and student actions

Mathematical discourse among students is central to meaningful learning of mathematics. Teachers carefully prepare and purposefully facilitate discourse, such as whole-class discussions that build on student thinking and guide the learning of the class in a productive disciplinary direction. Students are active members of the discourse community as they explain their reasoning and consider the mathematical explanations and strategies of their classmates. The actions listed in the table below provide some guidance on what teachers and students do as they engage in meaningful discourse in the mathematics classroom.

Facilitate meaningful mathematical discourse Teacher and student actions	
What are *teachers* doing?	**What are *students* doing?**
Engaging students in purposeful sharing of mathematical ideas, reasoning, and approaches, using varied representations. Selecting and sequencing student approaches and solution strategies for whole-class analysis and discussion. Facilitating discourse among students by positioning them as authors of ideas, who explain and defend their approaches. Ensuring progress toward mathematical goals by making explicit connections to student approaches and reasoning.	Presenting and explaining ideas, reasoning, and representations to one another in pair, small-group, and whole-class discourse. Listening carefully to and critiquing the reasoning of peers, using examples to support or counterexamples to refute arguments. Seeking to understand the approaches used by peers by asking clarifying questions, trying out others' strategies, and describing the approaches used by others. Identifying how different approaches to solving a task are the same and how they are different.

Pose Purposeful Questions

> *Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.*

Effective mathematics teaching relies on questions that encourage students to explain and reflect on their thinking as an essential component of meaningful mathematical discourse. Purposeful questions allow teachers to discern what students know and adapt lessons to meet varied levels of understanding, help students make important mathematical connections, and support students in posing their own questions. However, merely asking questions is not enough to ensure that students make sense of mathematics and advance their reasoning. Two critical issues must be considered—the types of questions that teachers ask and the pattern of questioning that they use.

Discussion

Researchers have created a variety of frameworks to categorize the types of questions that teachers ask (e.g., Boaler and Brodie 2004; Chapin and O'Connor 2007). Though the categories differ across frameworks, commonalities exist among the types of questions. For example, the frameworks generally include questions that ask students to recall information, as well as questions that ask students to explain their reasoning. Figure 14 displays a set of question types that synthesizes key aspects of these frameworks that are particularly important for mathematics teaching. Although the question types differ with respect to the level of thinking required in a response, all of the question types are necessary in the interactions among teachers and students. For example, questions that gather information are needed to establish what students know, while questions that encourage reflection and justification are essential to reveal student reasoning.

Question type		Description	Examples
1	Gathering information	Students recall facts, definitions, or procedures.	When you write an equation, what does the equal sign tell you? What is the formula for finding the area of a rectangle? What does the interquartile range indicate for a set of data?
2	Probing thinking	Students explain, elaborate, or clarify their thinking, including articulating the steps in solution methods or the completion of a task.	As you drew that number line, what decisions did you make so that you could represent 7 fourths on it? Can you show and explain more about how you used a table to find the answer to the Smartphone Plans task? It is still not clear how you figured out that 20 was the scale factor, so can you explain it another way?

Fig. 14. A framework for types of questions used in mathematics teaching

Question type		Description	Examples
3	Making the mathematics visible	Students discuss mathematical structures and make connections among mathematical ideas and relationships.	What does your equation have to do with the band concert situation? How does that array relate to multiplication and division? In what ways might the normal distribution apply to this situation?
4	Encouraging reflection and justification	Students reveal deeper understanding of their reasoning and actions, including making an argument for the validity of their work.	How might you prove that 51 is the solution? How do you know that the sum of two odd numbers will always be even? Why does plan A in the Smartphone Plans task start out cheaper but become more expensive in the long run?

Fig. 14. *Continued*

While the *types* of questions that teachers ask are important, so are the *patterns* of questions that they use during teacher-student interactions (Walsh and Sattes 2005). In the Initiate-Response-Evaluate (I-R-E) pattern, the teacher starts by asking a question to gather information, generally with a specific response in mind; a student responds; and then the teacher evaluates the response (Mehan 1979). It is not uncommon for teachers to allocate less than five seconds for a student to respond, and to take even less time to consider the answer themselves. This pattern of questioning generally affords very limited opportunities for students to think and provides teachers with no access to whether or how students are making sense of mathematics. Other questioning patterns involve more than asking recall questions. Two of these patterns of questioning are *funneling* and *focusing* (Herbel-Eisenmann and Breyfogle 2005; Wood 1998).

The funneling pattern of questioning involves using a set of questions to lead students to a desired procedure or conclusion, while giving limited attention to student responses that veer from the desired path. The teacher has decided on a particular path for the discussion to follow and leads the students along that path, not allowing students to make their own connections or build their own understanding of the targeted mathematical concepts. The I-R-E pattern is closely akin to funneling, though higher-level questions may be part of the funneling pattern.

In contrast, a focusing pattern of questioning involves the teacher in attending to what the students are thinking, pressing them to communicate their thoughts clearly, and expecting them to reflect on their thoughts and those of their classmates. The teacher who uses this pattern of questioning is open to a task being investigated in multiple ways. On the basis of content knowledge related to the topic and knowledge of student learning, the teacher plans questions and outlines key points that should become salient in the lesson.

wait time and immediately follows up with questions that become more directed toward one particular answer.

By contrast, the focusing example illustrates how the teacher purposefully blends all four types of questions. Some questions have been planned in advance of the lesson, along with consideration of possible student responses. Other questions are formulated on the spot, in response to student statements and actions during the lesson. Throughout the dialogue, the teacher strives to include questions that push students to clarify their ideas and make the mathematics visible, with the aim of deepening students' mathematical understanding in alignment with the intended learning goals.

Teacher and student actions

In effective teaching, teachers use a variety of question types to assess and gather evidence of student thinking, including questions that gather information, probe understanding, make the mathematics visible, and ask students to reflect on and justify their reasoning. Teachers then use patterns of questioning that focus on and extend students' current ideas to advance student understanding and sense making about important mathematical ideas and relationships. The teacher and student actions listed in the table below provide a summary of using questions purposefully in the mathematics classroom.

Pose purposeful questions **Teacher and student actions**	
What are *teachers* doing?	**What are *students* doing?**
Advancing student understanding by asking questions that build on, but do not take over or funnel, student thinking. Making certain to ask questions that go beyond gathering information to probing thinking and requiring explanation and justification. Asking intentional questions that make the mathematics more visible and accessible for student examination and discussion. Allowing sufficient wait time so that more students can formulate and offer responses.	Expecting to be asked to explain, clarify, and elaborate on their thinking. Thinking carefully about how to present their responses to questions clearly, without rushing to respond quickly. Reflecting on and justifying their reasoning, not simply providing answers. Listening to, commenting on, and questioning the contributions of their classmates.